

LOGARITHM & QUADRATIC EQUATION

$$\log_2 8 \rightarrow 8, 2 \text{ की Kitni degree है?}$$

$$\text{Ans} = 3 \rightarrow 2^3 = 8$$

$$\log_2 8 = 3$$

$$\log_4 64 \Rightarrow 64, 4 \text{ की Kitni degree है?}$$

$$4^3 = 64$$

$$\log_4 64 = 3$$

$$\text{Q1 } \log_{893} 893 = 1$$

$$(893)^1 = 893$$

$$\text{B) } \log_9 \frac{1}{81} = -2 \quad \frac{1}{9^2} = \frac{1}{81}$$

$$\text{(C) } \log_{97} 1 = 0 \quad (97)^0 = 1$$

$$\text{(D) } \log_{\sqrt{2}+1} (\sqrt{2}-1) = -1$$

$$\frac{(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)} = 1$$

$$\sqrt{2}-1 = \frac{1}{\sqrt{2}+1} = (\sqrt{2}+1)^{-1}$$

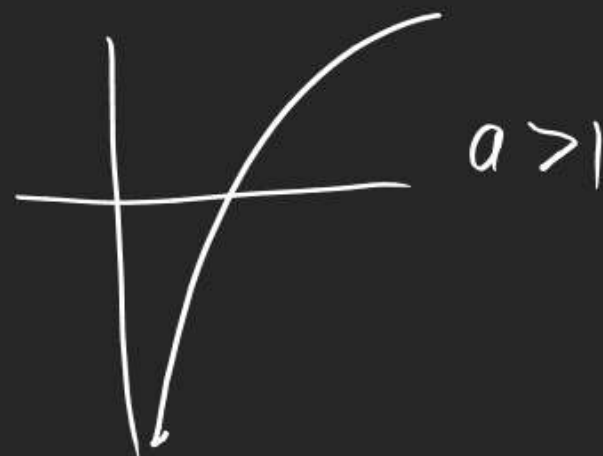
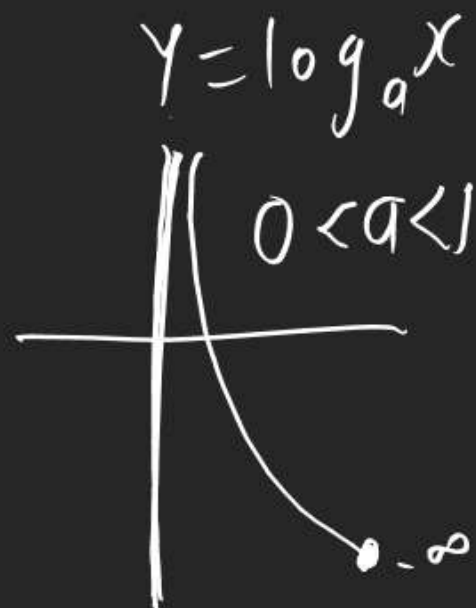
LOGARITHM & QUADRATIC EQUATION

3 Imp Basic Results

$$(1) \log_N N = 1 \quad ; \quad N > 0, N \neq 1$$

$$(2) \log_{\frac{1}{N}} N = -1 \quad ; \quad N > 0, N \neq 1$$

$$(3) \log_N 1 = 0 \quad ; \quad N > 0, N \neq 1$$



$$1) \log_{\frac{1}{2}} 0 = -\infty \text{ (Not Defined)}$$

$$2) \log_0 5 = \text{Not defined}$$

$$3) \log_1 5 = \text{Not defined}$$

$$4) \log_{-2} 8 = \text{Not defined}$$

$$5) \log_1 1 = \text{Not Defined}$$

LOGARITHM & QUADRATIC EQUATION

$$\log_2 (-8) = \text{Not defined}$$

-ve
not
Allowed

$$\log_1 8 = \text{Not defined}$$

Base = 1 Not Allowed

$$\log_0 8 = \text{Not defined}$$

Base 0 not Allowed

$$Q \quad Y = \log_4 (x-1) \text{ is defined for } x \in ?$$

$$\text{Base} = 4 > 0 \checkmark$$

$$\text{Base} = 4 \neq 1 \checkmark$$

$$x-1 > 0 \text{ Hina Padega}$$

$$x > 1$$

$$\text{Ans } \underline{x \in (1, \infty)}$$

1) Base has to be +ve.

2) It can not be 1

LOGARITHM & QUADRATIC EQUATION

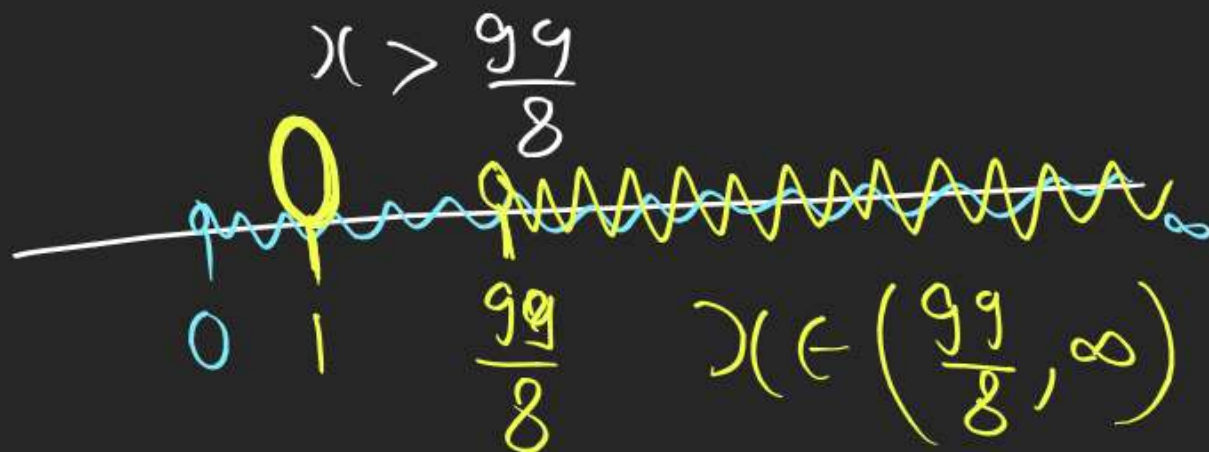
Q $y = \log_x (8x - 99)$ is defined for?

1) Base $= x > 0$

2) Base $= x \neq 1$

3) $8x - 99 > 0$

$x > \frac{99}{8}$



Q find a if $\log_3 (a^2 + 1) = 1$

$(a^2 + 1)$ 3 ki 1 deg hai

$3^1 = a^2 + 1$

$a^2 = 2$

$a = \pm\sqrt{2}$

(M2)

~~$\log_3 (a^2 + 1) = 1$~~

$a^2 + 1 = 3^1$

$a^2 = 2 \Rightarrow a = \pm\sqrt{2}$

LOGARITHM & QUADRATIC EQUATION

$$Q \log_{\tan 45^\circ} \tan 45^\circ = \log_{\textcircled{1}} 1 = \text{ND}$$

Base = 1 not Allowed

$$Q (\log_2 \tan 1^\circ) (\log_2 \tan 2^\circ) (\log_2 \tan 3^\circ) \dots 0 \dots (\log_2 \tan 89^\circ) = ?$$

Bilk Me Kahin

$$\log_2 \tan 45^\circ = \log_2 1 = 0$$

→ 0 is coming in Betⁿ & Inlith multiplication It will give 0

= 

LOGARITHM & QUADRATIC EQUATION

$$\begin{aligned} Q \log_{16}(\log_4(\log_2 16)) \\ \log_{16}(\log_4(4)) \\ \log_{16}(1) = 0 \end{aligned}$$

$$\begin{aligned} Q \log_8(\log_9(\log_3(3^3))) \\ \log_8(\log_9(3)) \\ \log_8\left(\frac{1}{2}\right) = -\frac{3}{2} \\ \left(\frac{1}{2}\right)^{-\frac{3}{2}} = 8 \end{aligned}$$

Q If x_1 & x_2 are Roots of Eqn.

$$\log_5\left(\log_{64}|x| + 25^x - \frac{1}{2}\right) = 2x$$

then $x_1 + x_2 = ?$ $8 + -8 = 0$

$$\begin{aligned} \log_{64}|x| + 25^x - \frac{1}{2} &= (5^2)^x \\ &= (5^2)^x \end{aligned}$$

$$\log_{64}|x| + \cancel{25^x} - \frac{1}{2} = \cancel{25^x}$$

$$\begin{aligned} \log_{64}|x| &= \frac{1}{2} \Rightarrow |x| = (64)^{\frac{1}{2}} = 8 \\ x &= \pm 8 \Rightarrow x_1 = 8, x_2 = -8 \end{aligned}$$

LOGARITHM & QUADRATIC EQUATION

$$a+b=7$$

$$a=3 \quad b=4$$

2 Imp Basic Properties

$$\log_m A + \log_m B = \log_m (A \cdot B)$$

$$\log_m A - \log_m B = \log_m \left(\frac{A}{B}\right)$$

Q If $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = 1 - a \log_7 2$

$$\log_{15} \log_{15} \sqrt[7]{15\sqrt{15\sqrt{15\sqrt{15}}}} = 1 - b \log_{15} 2$$

$$\log_7 \log_7 (7)^{\frac{7}{8}}$$

$$\log_7 \left(\frac{7}{8}\right) = \log_7 7 - \log_7 8 = 1 - \log_7 2^3$$

$$\log_{15} \log_{15} (15)^{\frac{15}{16}} = 1 - 3 \log_{15} 2$$

$$\log_{15} \left(\frac{15}{16}\right) = \log_{15} 15 - \log_{15} 16 = 1 - 4 \log_{15} 2$$

$$\sqrt{7} = 7^{\frac{1}{2}}$$

$$\sqrt{7\sqrt{7}} = \sqrt{7 \times 7^{\frac{1}{2}}} = \sqrt{7^{\frac{3}{2}}}$$

$$\sqrt{7\sqrt{7\sqrt{7}}} = \sqrt{7 \times 7^{\frac{3}{4}}} = \sqrt{7^{\frac{7}{4}}} = 7^{\frac{7}{8}}$$

$\log_m AB$
 $\log_m \frac{A}{B}$
 $B \log_m A$

$$Q \log_2(4^{x+1} + 4) \cdot \log_2(4^x + 1) = \log_{\frac{1}{\sqrt{2}}} \left(\sqrt{\frac{1}{2}} \right)$$

$$\log_2(4^x \cdot 4 + 4) \cdot \log_2(4^x + 1) = \log_{\frac{1}{\sqrt{2}}} \left(\frac{1}{2} \right)^{\frac{1}{2}}$$

find x?

$$\log_2(4 \cdot (4^x + 1)) \cdot \log_2(4^x + 1) = 3$$

$$(\log_2 4 + \log_2(4^x + 1)) \cdot \log_2(4^x + 1) = 3$$

$$(2 + \log_2(4^x + 1)) \cdot \log_2(4^x + 1) = 3$$

$$(2 + t)t = 3 \Rightarrow t^2 + 2t - 3 = 0$$

$$(t+3)(t-1) = 0$$

$$t = -3 \text{ \& } t = 1$$

$$t = -3 \text{ \& } t = 1$$

$$\log_2(4^x + 1) = -3$$

$$4^x + 1 = 2^{-3}$$

$$4^x + 1 = \frac{1}{8}$$

$$4^x = \frac{1}{8} - 1 = -\frac{7}{8}$$

Ex b f x n

⊕

x = ∅

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$$\log_2(4^x + 1) = 1$$

$$4^x + 1 = 2^1$$

$$4^x = 1 = 4^0$$

$$\boxed{x = 0} \checkmark$$

✓

Always check

Ur Ans by putting
value of x in Qs.

$$\log_2(4^1 + 4) \cdot \log_2(4^1 + 1) = 3$$

$$\log_2 8 \cdot \log_2 2 = 3 \Rightarrow 3 \times 1 = 3 \checkmark$$

$$x+1) - \log_{\frac{1}{2}}(x+1) = 0$$

$$A - \log B = \log A/B$$

$$\frac{(2x+1)}{(x+1)}$$

$$\frac{2x+1}{x+1} = \left(\frac{1}{2}\right)^0 = 1$$

$$2x+1 = x+1$$

$$x=0 \checkmark$$

$$\text{Q If } \log_2(\log_3(\log_4 x)) = 0 \mid 2^0 = 1$$

$$\& \log_3(\log_4(\log_2 y)) = 0$$

$$\& \log_4(\log_2(\log_3 z)) = 0$$

$$\text{then } x+y+z = ?$$

$$\log_2(\log_3(\log_4 x)) = 0$$

$$\log_3(\log_4 x) = 2^0 = 1$$

$$\log_4 x = 3^1 = 3$$

$$x = 4^3 = 64$$

$$\log_4(\log_2 y) = 3^0 = 1$$

$$\log_2 y = 4^1 = 4$$

$$y = 2^4 = 16$$

$$\log_2(\log_3 z) = 1$$

$$\log_3 z = 2^1$$

$$z = 3^2 = 9$$

$$64 + 16 + 9 = 89$$

BCT

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}$$

Q Real value of x for which

$$\log_6 9 - \log_9 27 + \log_6 x = \log_{64} x - \log_6 4$$

hold true

$$\Rightarrow \log_6 9 - \log_{3^2} 3^3 + \log_6 x - \log_{64} x + \log_6 4 = 0$$

$$\Rightarrow \log_6 36 \cdot x = \log_9 27 + \log_{64} x$$

$$\log_6 36 + \log_6 x = \frac{3}{2} + \log_{2^6} x$$

$$2 + \log_6 x = \frac{3}{2} + \frac{1}{6} \log_2 x$$

$$\frac{1}{2} = \frac{1}{6} \log_2 x - \frac{\log_2 x}{\log_2 6}$$

$$= \frac{3}{2} \times 1 = \frac{3}{2}$$

$$\log_m A^B = B \log_m A$$

$$\log_m^B A = \frac{1}{B} \log_m A$$

$$\log_9 27 = \log_{3^2} 3^3$$

$$= \frac{3}{2} \left[\log_{\frac{3}{3}} \frac{3}{3} \right]$$

LOGARITHM & QUADRATIC EQUATION

Base change Theorem

$$\log_a m = \frac{\log_b m}{\log_b a}$$

$$\log_6 x = \frac{\log_2 x}{\log_2 6}$$

$$\log_9 x = \frac{\log_2 x}{\log_2 9} = \frac{\log_{13} x}{\log_{13} 9}$$

$$\frac{1}{2} = \frac{1}{6} \log_2 x - \frac{\log_2 x}{\log_2 6}$$

$$\frac{1}{2} = \log_2 x \left\{ \frac{1}{6} - \frac{1}{\log_2 6} \right\}$$

$$\frac{1}{2} = \log_2 x \left\{ \frac{\log_2 6 - 6}{6 \log_2 6} \right\}$$

$$\frac{6 \log_2 6}{(\log_2 6 - 6)} \times \frac{1}{2} = \log_2 x$$

Hold

$$\begin{aligned}
 Q \log_2(4-x) &= 4 - \log_2(-2-x) \xrightarrow{x=-4} \log_2(4+4) = 4 - \log_2(-2+4) \\
 \log_2(4-x) + \log_2(-2-x) &= 4 \quad \text{find } x \\
 \log_2((4-x)(-2-x)) &= 4 \\
 (4-x)(-2-x) &= 2^4 = 16 \\
 x^2 - 2x - 8 &= 16 \\
 x^2 - 2x - 24 &= 0 \\
 (x-6)(x+4) &= 0 \\
 x &= -4, 6 \quad \text{✓, } \otimes
 \end{aligned}$$

$$\begin{aligned}
 x &= 6 \\
 \log_2(4-6) &= 4 - \log_2(-2-6) \\
 \otimes & \qquad \qquad \otimes
 \end{aligned}$$

$$\boxed{x = -4}$$

LOGARITHM & QUADRATIC EQUATION

$$Q \log_{10} \left(\frac{a+b + \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) + \log_{10} \left(\frac{a+b - \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) = \log_{10}(x)$$

find x?

$$\log_{10} \left\{ \left(\frac{a+b + \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) \left(\frac{a+b - \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) \right\}$$

$\log A + \log B$
 $\frac{A+B}{2}$

$$\log_{10} \left\{ \frac{a^2 + b^2 - ((a+b)^2 - 4(a+b))}{4} \right\} = \log_{10} x$$

$$\log_{10}(a+b) = \log_{10} x \Rightarrow \boxed{x = a+b}$$